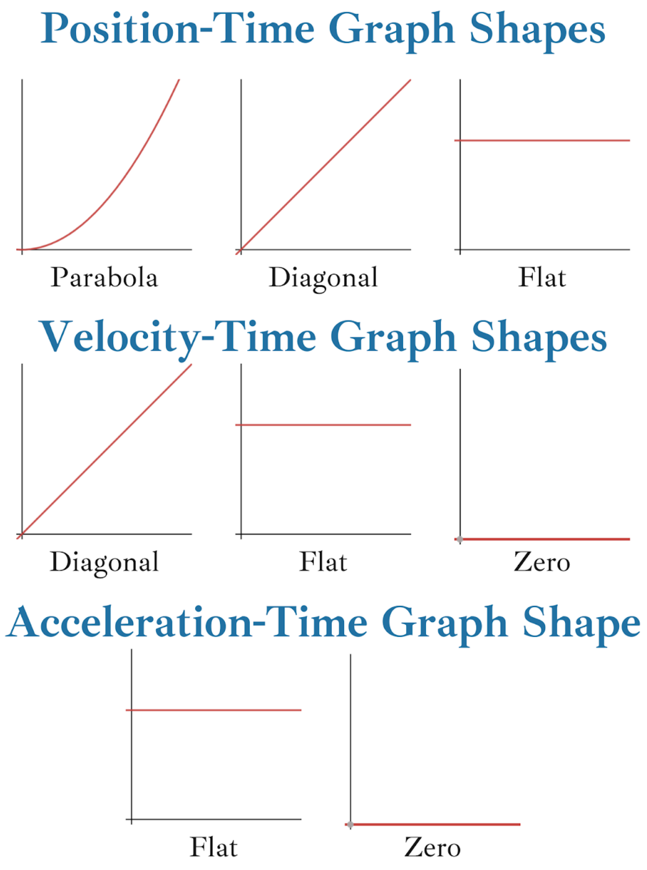
Forces and Motion #2

York College & University Centre

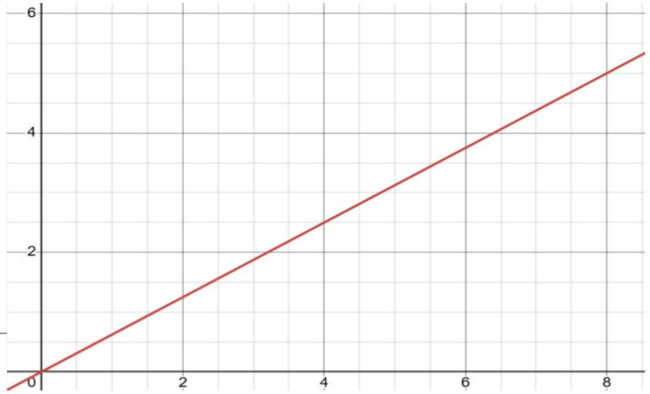
Motion Graphs

- There are 3 main types of motion graph:
  - Displacement-Time
  - Velocity-Time
  - Acceleration-Time

- We can interchange between values using our graphs:
  - Gradient of a displacement–time graph → velocity
  - Gradient of a velocity–time graph → acceleration
  - Area under a velocity–time graph → displacement
  - Area under an acceleration–time graph → change in velocity



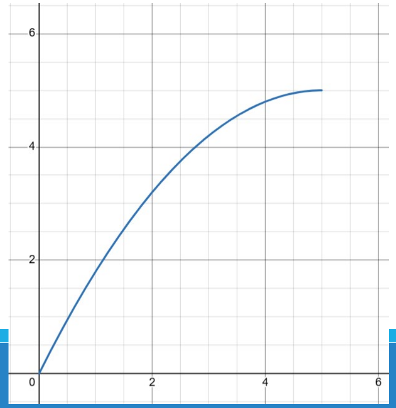
Motion Graphs – Straight



- **A straight-line motion graph indicates either velocity and/or acceleration is constant**

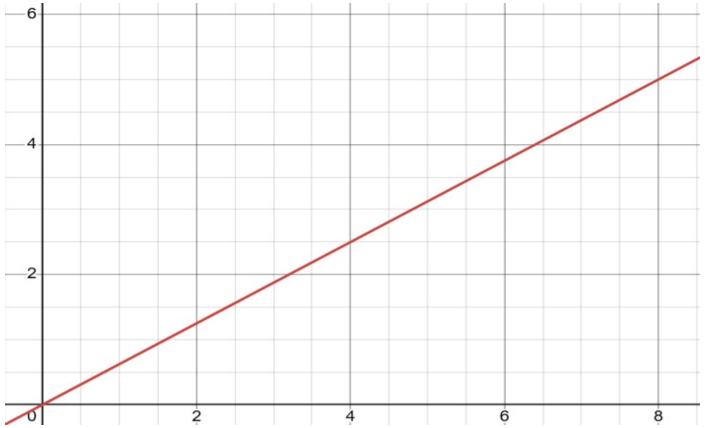
- **This means that velocity and/or acceleration doesn't change as time changes:**
  - Straight-line displacement-time graph → constant velocity → 0 acceleration
  - Straight-line velocity-time graph → constant acceleration

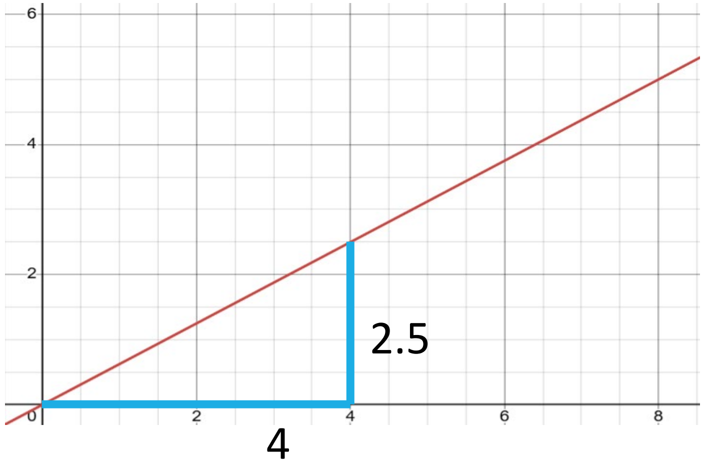
- **When we convert a straight-line graph in the opposite direction, we get a curve:**
  - Straight-line acceleration-time graph → Quadratic velocity-time graph → Cubic displacement-time graph
  - Straight-line velocity-time graph → Quadratic displacement-time graph



Motion Graphs – Gradient (Straight)

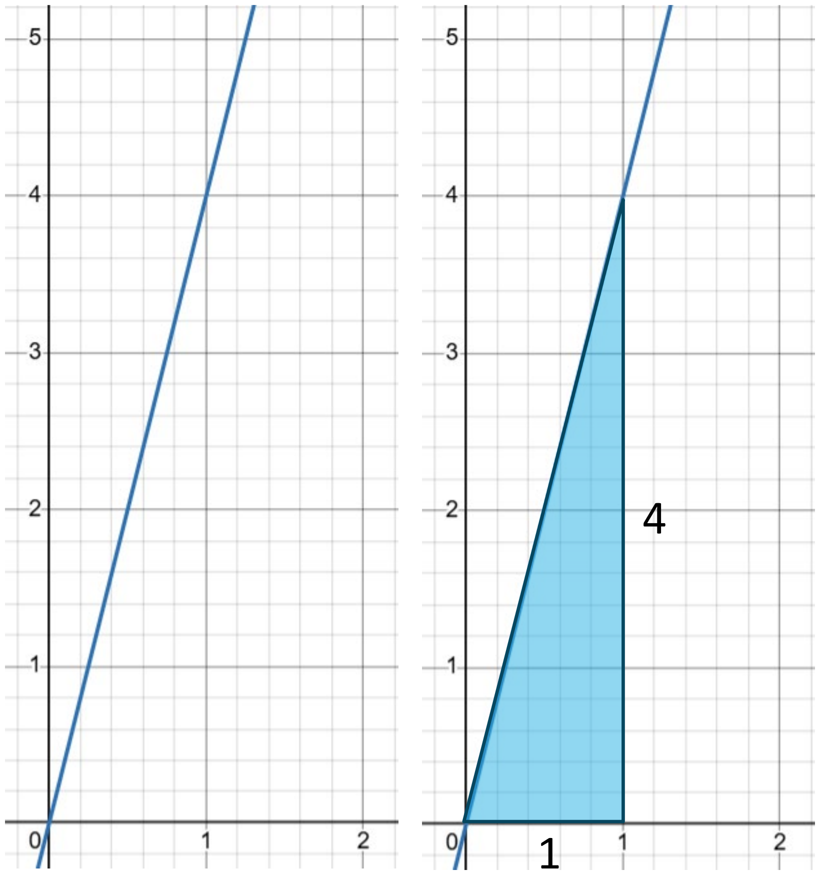
- To work out the velocity (gradient) of the **straight-line** displacement-time on this slide we do:
- $\frac{change\ in\ y}{change\ in\ x}$
- So we do
- $\frac{2.5}{4} = 0.625$
- So our velocity is $0.625\ m/s$





Motion Graphs – Area (Straight)

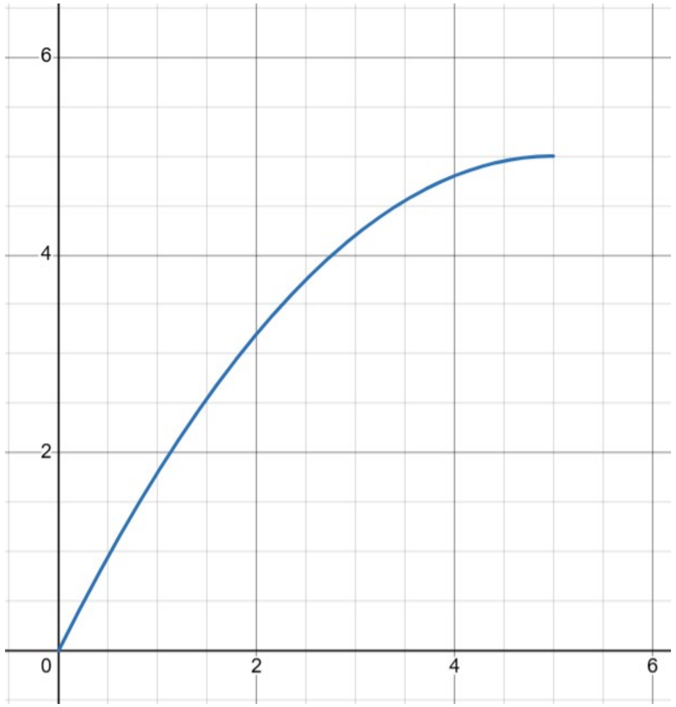
- To work out the displacement (area under the graph) of the **straight-line** velocity-time on this slide we do:
- $\frac{1}{2}(change\ in\ x * change\ in\ y)$
- So we do
- $\frac{1}{2}(4 * 1)$
- So our displacement is $2m$



Motion Graphs – Curve

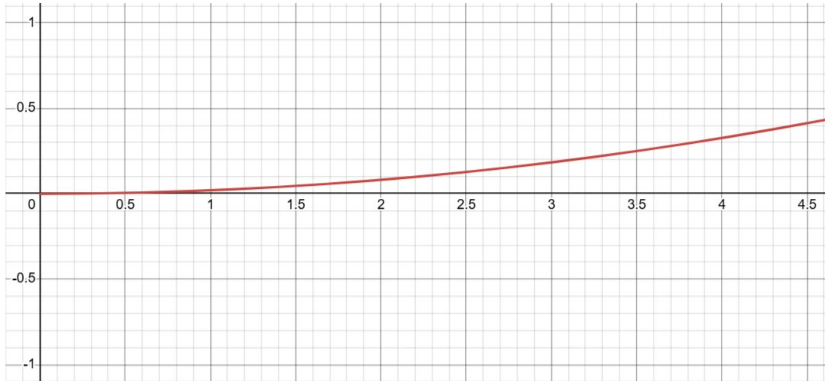
- **Our graphs won't always be straight line though as displacement and velocity isn't always linear, so we sometimes must use differentiation/integration**

- If you are asked to work out a value from a curve you will be given its equation



Motion Graphs – Gradient (Curve)

- $\frac{d}{dx}[c * f(x)] = c * \frac{d}{dx}[f(x)]$



- To work out the velocity when $t = 3$ from the displacement-time graph on this slide that is defined as $s = \frac{t^2}{49}$ we use differentiation

- First we need to differentiate the equation so we follow the derivative rule, making:

$$s = \frac{t^2}{49} \rightarrow v = \frac{2t}{49}$$

- Then we just substitute in the t value $\mathrm{v} = \frac{2(3)}{49}$ which gives us $\mathrm{v} = \frac{6}{49} = 0.122$

- So our velocity is 0.122 @ 3s

Motion Graphs – Area (Curve)

- $\int ax^n dx = a * \frac{x^{n+1}}{n+1} + c$

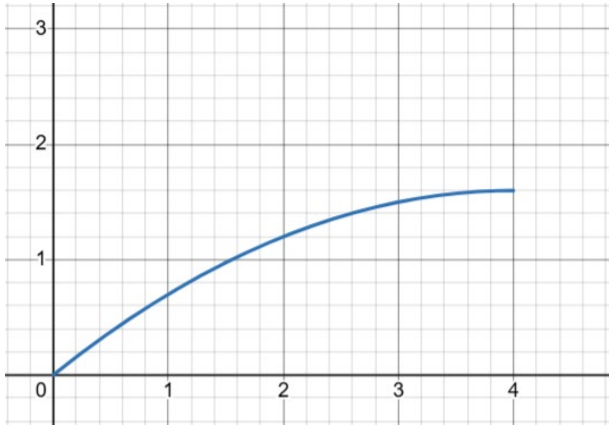
- To work out the displacement $(0 \le t \le 4)$ based on the velocity-time graph on this slide (defined as: $v = 0.8t - 0.1t^2$) we want to use integration

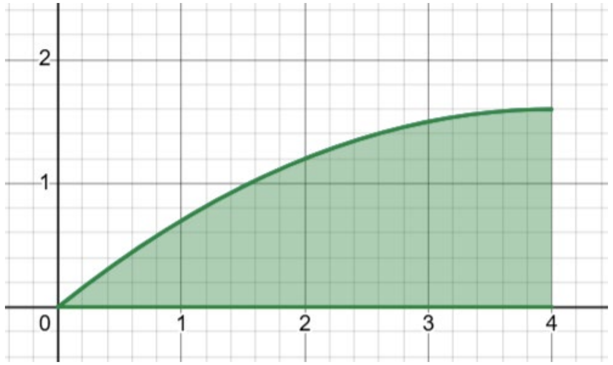
- First we need to integrate the equation so we follow the integration rule, making:

$$v = 0.8t - 0.1t^2 \rightarrow s = \frac{0.8}{2}t^2 - \frac{0.1}{3}t^3 \rightarrow s = 0.4t^2 - \frac{1}{30}t^3$$

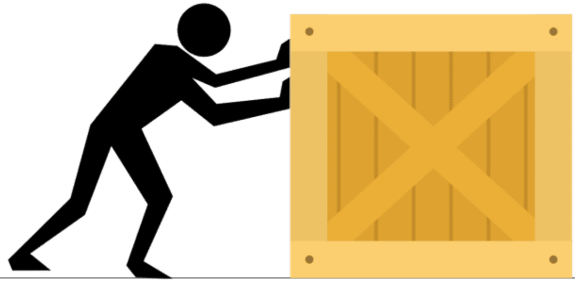
- Then we just substitute in the minimum and maximum value of t and take them away from each other:

$$s = \left(0.4(4)^2 - \frac{1}{30}(4)^3\right) - \left(0.4(0)^2 - \frac{1}{30}(0)^3\right) = 4.2667 - 0 = 4.2667m$$

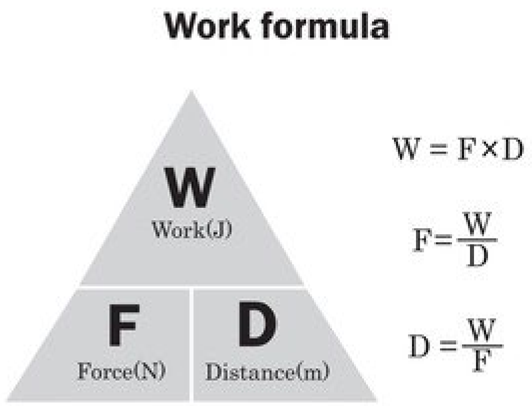




Work Done



- Work is done when a force causes an object to move through a distance.
- Work transfers energy from one system to another.
- If an object does not move, no work is done.
- We can calculate work using the equation:
- $\boldsymbol{W = F * s}$
- Where:
- W = Work done (in Joules)
- F = Force applied
- s = distance moved in the direction of force



Kinetic Energy

- Any moving object has kinetic energy

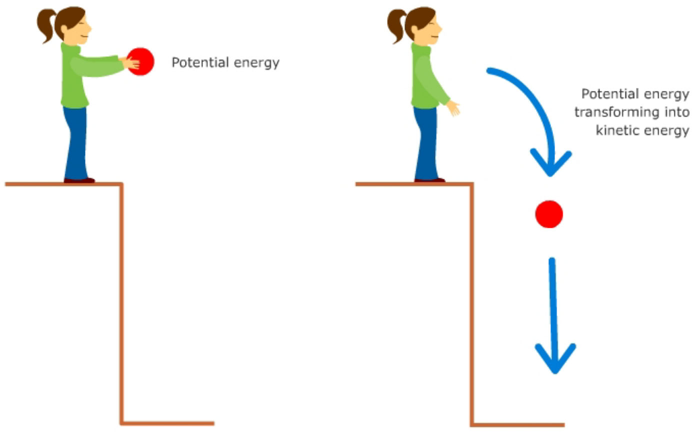
- Velocity has the largest effect because it is squared.

- We can calculate the kinetic energy of a moving object using:

- $\boldsymbol{K_e = \frac{1}{2}mv^2}$

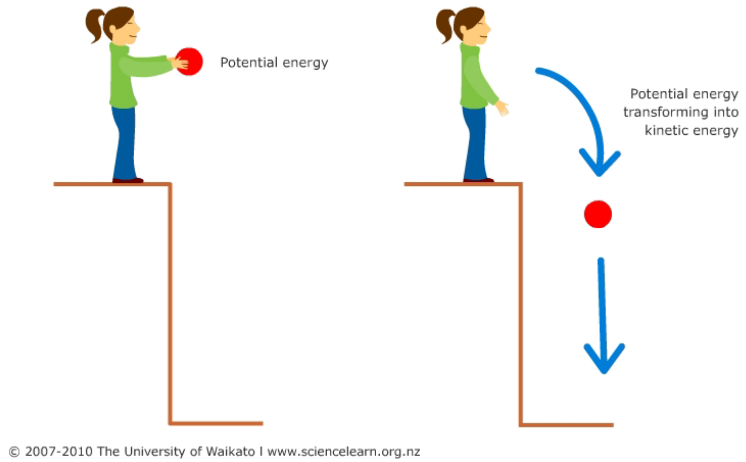
- Where:

- K_e = Kinetic energy (in Joules)
- m = mass
- v = velocity(speed)



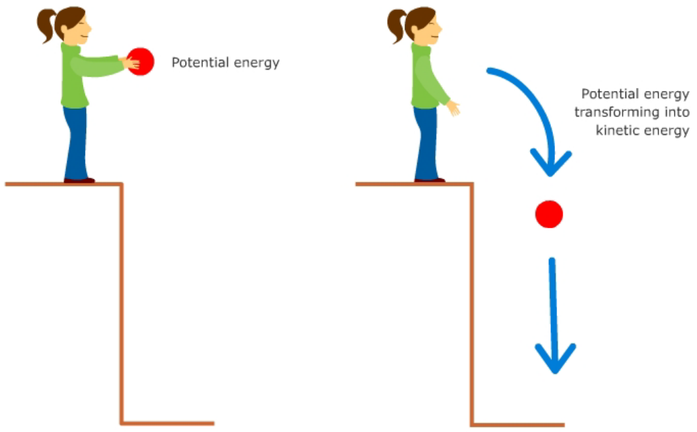
Kinetic energy and Work done

- When something is moving this is because it has had work done on it
- When work is done energy in one form is transferred to the kinetic energy of the moving object.
- To stop the object again, the same amount of work would have to be done to bring it back to rest.



Gravitational Potential Energy

- Any object above the earth's surface has gravitational potential
- When the object falls, potential energy converts to kinetic energy.
- We can calculate the gravitational potential of an object using:
- $E_p = mgh$
- Where:
- E_p = Gravitational potential (in Joules)
- m = mass
- g = Effect of gravity (9.81)
- h = height above the earth's surface



Gravitational potential energy and Work done

- If an object is lifted, work is done against the force of gravity.
- When work is done energy is transferred to the object and it gains gravitational potential energy.
- If the object falls from that height, the same amount of work would have to be done by the force of gravity to bring it back to the Earth's surface.